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JOBS FOR THE FUTURE

DEEPER LEARNING RESEARCH SERIES

DEEPER TEACHING

By Magdalene Lampert
December 2015

EDITORS' INTRODUCTION TO
THE DEEPER LEARNING RESEARCH SERIES

In 2010, Jobs for the Future—with support from the Nellie Mae Education Foundation—launched the Students at the Center initiative, an effort to identify, synthesize, and share research findings on effective approaches to teaching and learning at the high school level.

The initiative began by commissioning a series of white papers on key topics in secondary schooling, such as student motivation and engagement, cognitive development, classroom assessment, educational technology, and mathematics and literacy instruction.

Together, these reports—collected in the edited volume *Anytime, Anywhere: Student-Centered Learning for Schools and Teachers*, published by Harvard Education Press in 2013—make a compelling case for what we call "student-centered" practices in the nation's high schools. Ours is not a prescriptive agenda; we don't claim that all classrooms must conform to a particular educational model. But we do argue, and the evidence strongly suggests, that most, if not all, students benefit when given ample opportunities to:

> Participate in ambitious and rigorous instruction tailored to their individual needs and interests

> Advance to the next level, course, or grade based on demonstrations of their skills and content knowledge

> Learn outside of the school and the typical school day

> Take an active role in defining their own educational pathways

Students at the Center will continue to gather the latest research and synthesize key findings related to student engagement and agency, competency education, and other critical topics. Also, we have developed—and have made available at www.studentsatthecenterhub.org—a wealth of free, high-quality tools and resources designed to help educators implement student-centered practices in their classrooms, schools, and districts.

Further, and thanks to the generous support of The William and Flora Hewlett Foundation, Students at the Center has expanded its portfolio to include an additional and complementary strand of work.

The present paper is part of our new series of commissioned reports—the Deeper Learning Research Series—which aim not only to describe best practices in the nation's high schools but also to provoke much-needed debate about those schools' purposes and priorities.

In education circles, it is fast becoming commonplace to argue that in 21st—century America, each and every student must aim for "college, career, and civic readiness." However, and as David T. Conley described in the first paper in this series, a large and growing body of empirical research shows that we are only just beginning to understand what "readiness" really means. Students' command of academic skills and content certainly matters, but so too does their ability to communicate effectively, to work well in teams, to solve complex problems, to persist in the face of challenges, and to monitor and direct their own learning—in short, the various kinds of knowledge and skills that have been grouped together under the banner of "deeper learning."

What does all of this mean for the future of secondary education? If "readiness" requires such ambitious and multi-dimensional kinds of teaching and learning, then what will it take to help students become genuinely prepared for life after high school, and what are the implications for policy and practice?

We are delighted to share this installment in the Deeper Learning Research Series, and we look forward to the conversations that all of these papers will provoke.

To download the papers, executive summaries, and additional resources, please visit the project website: www.jff.org/deeperlearning.

Rafael Heller, Rebecca E. Wolfe, Adria Steinberg

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Jobs for the Future works with our partners to design and drive the adoption of education and career pathways leading from college readiness to career advancement for those struggling to succeed in today's economy. We work to achieve the promise of education and economic mobility in America for everyone, ensuring that all low-income, underprepared young people and workers have the skills and credentials needed to succeed in our economy. Our innovative, scalable approaches and models catalyze change in education and workforce delivery systems.

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Students at the Center

JOBS FOR THE FUTURE

Students at the Center—a Jobs for the Future initiative—synthesizes and adapts for practice current research on key components of student-centered approaches to learning that lead to deeper learning outcomes. Our goal is to strengthen the ability of practitioners and policymakers to engage each student in acquiring the skills, knowledge, and expertise needed for success in college, career, and civic life. This project is supported generously by funds from the Nellie Mae Education Foundation and The William and Flora Hewlett Foundation.

WWW.STUDENTSATTHECENTER.ORG

ABOUT THE AUTHOR

Magdalene Lampert advises the Boston Plan for Excellence on adult learning systems in its Teaching Academies and on the design and development of the Boston Teacher Residency's clinical teacher education/induction program. She also consults with New Visions for Public Schools in New York City to design teacher development that supports students' achievement of the learning goals in the Common Core State Standards. From 2007 until 2011, she coordinated *Learning Teaching in, from, and for Practice,* a project for developing teacher education pedagogy across the University of Washington, the University of Michigan, and the University of California, Los Angeles (UCLA). Dr. Lampert has taught elementary and high school mathematics, pre-service and in-service teacher education, and doctoral courses for aspiring teacher educators. She has written extensively about teaching practice, including the book *Teaching Problems and the Problems of Teaching.* Dr. Lampert is Professor Emerita in the University of Michigan School of Education, where she held the George Herbert Mead Chair in Education. She received the 2014 Outstanding Contribution to Education Award from the Harvard Graduate School of Education and the 2014 AACTE Outstanding Journal of Teacher Education Article Award from the American Association of Colleges for Teacher Education.

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INTRODUCTION

"What sort of endeavor is teaching?" The answer seems simple: One in which knowledge and skills are transmitted.

All true, but not all that is true. One might also say that teachers try to improve their students' minds, souls, habits.

−David K. Cohen (2011), from *Teaching and its Predicaments* (p. 4)

The Hewlett Foundation defines deeper learning as "an umbrella term for the skills and knowledge that students must possess to succeed in 21st-century jobs and civic life" (Hewlett Foundation 2013). Under that umbrella fit a set of competencies that "students must master in order to develop a keen understanding of academic content and apply their knowledge to problems in the classroom and on the job." These competencies include: master core academic content; think critically and solve complex problems; work collaboratively; communicate effectively; learn how to learn; and develop academic mindsets.

What do these deeper learning competencies imply for the work of teaching? What does one do in a classroom to move students' minds, souls, and habits in their direction? This paper addresses these questions.

What Would Teaching for Deeper Learning Need to Accomplish?

Most students in most secondary schools are accustomed to learning in two ways: by listening to the teacher and reading books and other texts (Mehta & Fine 2015; Cusick 1983; Graff 2003). In a sense, these familiar ways of learning or "doing school" work for them because what they are expected to know and be able to do tends to be intellectually shallow (Jennings 2012; Cawelti 2006). However, if they are going to be expected to meet deeper learning expectations, their everyday experiences in the classroom will have to look quite different.

First, "mastering core academic content," as Hewlett defines it, involves more than remembering terms, facts, dates, and formulas. It means understanding "key principles and relationships within a content area and organiz[ing]

information in a conceptual framework." Putting that together with the expectation that students should be able to "think critically and solve problems" suggests a more active and personal intellectual process than the familiar scholastic exercises of copying, memorizing, and reproducing a framework developed by someone else (Bransford et al. 2000). This process calls for what Cohen refers to as "minds at work" (Cohen 2011).

This, then, is the first set of questions we need to ask if we want reconsider what we mean by teaching in light of the goals of deeper learning: How, exactly, does one teach a "conceptual framework" or instruct students on how to construct conceptual frameworks of their own? *Whose* way of organizing information in a domain do we choose to teach? Should we ask students to organize information into their own unique categories and relationships? If so, should they do that work independently or with our guidance?

Second, Hewlett's definition of deeper learning requires that students learn "communication and collaboration," which means they cannot just work on their own. Rather, they need to "reason critically and solve problems" in the company of others doing the same activities. This also affects how we think about the work of teaching. Communication and collaboration are best learned in what some scholars call "a community of practice," where shared norms and common ways of defining problems (and the nature of solutions) are fundamental to learning (Weick & McDaniel 1989; Wenger 1998; Rogoff et al. 1995). We have to ask, then: What does a teacher do to get students to interact with one another in these ways? How much preparation does a teacher do before students communicate and collaborate, and to what extent do teachers just improvise?

If I wanted to learn, say, the names of the countries involved in the 20th-century world wars, I might study a list I had been "taught" by a reliable book or teacher. If I then succeed in reciting or writing down that information when asked to do so, I might be justified in saying that I had "learned" it.

But to learn how to think critically, collaborate, and communicate requires an entirely different approach. Learning *how to do* these things requires that I actually *do them*, just as *learning how to swim* requires that I *swim* (however badly, at first). "Learning" here is both the goal and the means of getting to the goal (Sfard 2008). It requires not just a mind at work but a mind working on meaningful tasks, in concert with others.

In order to solve complex problems collaboratively, I need to practice solving them, expose my solutions to others' scrutiny, and persist in trying various approaches. I won't learn to solve problems collaboratively by reading books or listening to lectures, though books and lectures about collaborative problem solving might be useful resources. To become skilled at communicating effectively, I need to express my ideas in spoken or written language; if my audience doesn't "get" what I am trying to say or write, then I need to figure out how to revise it and try again (Staples & Truxaw 2010; Horn 2012). In taking on the task of convincing others to change their minds, I need to open myself up to social risks that are not usually associated with schoolwork. If I am the teacher, I need to ask: In order to get students to reason, collaborate, and communicate with one another, what kinds of instruction are needed? How can teachers be of assistance?

Finally, deeper learning is as much about who we want students to *become*—intellectually, at least—as it is about what we want them to *possess*. According to Hewlett, "learning how to learn" (the fifth competency) requires "caring about the quality of one's work, enjoying and seeking out learning on your own and with others." This has more to do with commitment and interest than skills and knowledge per se. Similarly, to "develop academic mindsets" means that students:

> Develop positive *attitudes and beliefs about themselves* as learners that increase their academic perseverance and prompt them to engage in productive academic behaviors [so that they are] *committed to* seeing work

through to completion, meeting their goals and doing quality work, and thus search for solutions to overcome obstacles (Hewlett Foundation 2013).

It goes without saying that such desires, attitudes, and beliefs are shaped, at least in part, by the kinds of interactions students have with peers and adults, both in and out of school (Roeser et al. 2000; Wortham 2004; Gordon 2000; Greeno 2001). Students learn how to learn and shape their identities with regard to academic work by participating in many such exchanges over time.[1] But what, exactly, does teaching look like in these exchanges? And how do teachers persuade students to invest more of their time, and more of *themselves*, in their academic work?

Given what we know about U.S. adolescents, it seems reasonable to assume that for most high school students, it will be impossible to build deeper learning competencies without simultaneously changing what they care about and, in a sense, who they are (Eckert 1989, 1990).[2] For example, in the 2009 Survey of Student Engagement (Indiana University, Bloomington), 62 percent of those surveyed said that their "top priority" was to play video games, surf the web, talk on the phone, or socialize outside of school, and the daily number of hours they reported spending on these activities was roughly proportional to the importance they assigned them. In short, teaching for deeper learning will need to support a kind of "identity transformation," as well as providing students with different ways to learn different kinds of content. The question is, how does a teacher support students in building new identities, changing their priorities, and choosing to dig into academic content?

If learning deeply requires students to be so bold as to change the ways in which they identify themselves and their interests, then teachers must support the risks involved in that change, as well. And if learning requires certain kinds of social interaction, then teaching needs to structure that interaction to provide the experiences that shape a student's scholarly self and make it safe to venture into new territory.

The teaching that results from answering the questions above will not be teaching as we have known it over the past century or longer (Cohen 1988, 2011; Cuban 1993). Perhaps all teaching changes students' minds, souls, and habits, as Cohen claims, but teaching for deeper learning does so deliberately, with specific and well-articulated ends.

TWO WAYS OF TEACHING: A COMPARISON

In order to determine whether teaching or learning is "deep," we first must be sure that the teacher and students are working on material that is in fact worth learning. So I begin this section by introducing a bit of the "core content" that is widely understood to be central to the secondary school curriculum. I then describe—based on my own classroom observations—how two teachers taught that content and what students did to learn it. While I focus on the details of these two lessons, one in a classroom focused on deeper learning, the other not, it will become clear that deeper teaching extends well beyond the lesson itself. It requires us to base every interaction between teachers and learners on a new understanding of what it means to teach and learn.

The first lesson typifies the most common sort of instruction in secondary schools across the U.S. (Mehta & Fine 2015; Cusick 1983, 1973; Graff 2003). The second features a teacher who is at the same point in the curriculum, teaching the same subject, but trying to practice what I call "deeper teaching" to support students' deeper learning. Such teaching is, comparatively, both intellectually and socially ambitious (Cohen 2011; Lampert & Graziani 2009; Newmann & Associates 1996). Over the next several pages, I put these two lessons under a microscope, focusing on moment-by-moment exchanges between the teachers and their students, in order to call precise attention to what it means to teach deeply, how such teaching differs from the norm, and what sorts of knowledge and skills it requires. I conclude by offering some thoughts on what it will take to promote and sustain such deeper teaching on a large scale.

I suspect that the interactions between teacher and students in the first teacher's lesson will seem entirely familiar to most readers. After all, these are the activities that many of us picture when we imagine a typical classroom: the teacher talks, usually to the whole class from the front of the room; the talk "explains" facts or procedures; and the class, sitting in desks facing the teacher, takes notes on what the teacher says or writes on the board.[3] If students talk at all, they provide answers to the teacher's questions, which the teacher judges to be correct or incorrect (Sims 2008; Holt 1969). This form of teaching has persisted in the U.S. for more than a century (Cohen 1985; Cuban 1993; Mehta & Fine 2015). Individual teachers don't invent it—it is a re-enactment of the way they were taught (Lortie 1975; Sam & Ernest 2000; Barkatsas, Tasos & Malone 2005), and it is what many students and parents expect of them.

The second lesson presents a very different picture. Students talk to one another, collaborating and communicating as they make sense of a complex problem. The teacher moves and talks in ways that engage students publicly as learners, listening to and representing their thinking for consideration by the whole class.

> The teacher moves and talks in ways that engage students publicly as learners, listening to and representing their thinking for consideration by the whole class.

Although some teachers may find their own way to a form of deeper teaching–perhaps seeing themselves as mavericks, standing apart from their peers–the teacher in this example did not invent deeper teaching any more than the teacher in the first example invented her approach. Rather, her lesson was developed as a part of a purposeful effort to design instruction and professional learning that deliberately organizes interactions between teachers and learners to bring about the kinds of competencies that characterize deeper learning.

Both of the lessons are more or less consistent with the ways in which these two teachers taught throughout the school year, and they are representative of the regular patterns of interaction–what some scholars call the "culture of instruction"–that characterized each classroom (Deal & Peterson 1999, 2009).

By participating in such a culture over time, individuals in a classroom environment learn the answers to basic questions about what it means to be a learner and a teacher (e.g. How are teachers supposed to act? How are students supposed to act? What are teachers and students supposed to say to one another? What are they supposed to believe? How are they supposed to relate to one another?) (Goos 2004; Horn 2012). Participating in a classroom culture is also how students learn what they are "good at" and whether or not that includes academics. In other words, classroom interactions–especially the shared culture, norms, and language that their teachers help them to create–develop each student's "mindset" about whether they can, or want to, succeed in school.
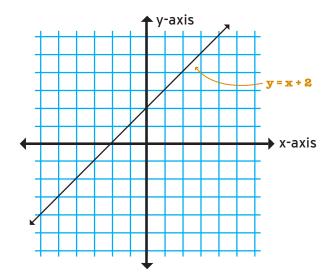
An Example of "Core Content"

From within the current subject-based organization of secondary schools, I have chosen to discuss two mathematics lessons. By writing from this perspective, I am not taking a stand for or against "siloed" school organization, where math is taught separately from science, history and literature. Instead, I merely want to suggest that deeper learning, and deeper teaching, are possible within the familiar curricular framework, and that curriculum in turn can be relevant to solving complex, on-the-job problems in many fields.

I focus here on algebra, which is usually taught as a distinct subject area sometime between eighth and tenth grade (though some elements of algebraic thinking may be learned in earlier grades). Studying algebra can provide

an introduction to a powerful mathematical language that people use to describe patterns and make predictions. It can be an opportunity to learn how to learn in new ways. It can open up access to social and economic resources (Moses & Cobb 2001). Or it can be an exercise in memorizing formulas and rules, as is the case in much U.S. education.

For students to achieve mastery of the core content of algebra, they need to study functions, i.e., mathematical relationships in which one quantity changes in relation to another. Here, I look at two lessons that have to do with the *rate of change of a linear function*. Mastering the concept of rate of change (sometimes referred to as *slope*)[4] enables us to think productively about problems like how to finance an expensive purchase, determine the safest gradient for a road, or design an engine. It can be a useful tool for making important decisions both in work and in civic life.[5] The news is full of stories whose meaning can only be grasped by seeing how one event is a function of another (Ritchhardt 1997). Students who understand functions and their rates of change can go on to study calculus, the mathematical gateway to engineering, medicine, economics, information technology, and many other fields.

When they study rate of change, students begin by working with linear functions, so named because represented on a coordinate graph[6] they look something like this:

The line on this graph tells us that no matter what quantity we start with ("x")–whether it is expressed as a whole number or a fraction, whether it is greater or less than zero–the related quantity ("y") is always two units larger. For this kind of function, it is possible to extrapolate

important information about how y changes in relation to x from just two pieces of data: the slope of the line and the point where it crosses the vertical line that goes through the origin (called the "y-intercept"). We can predict what y will be for any x, no matter how great or tiny the quantity. The lines on the graph above have arrows on their ends because they represent only a small part of infinite lines reaching in both directions.

Studying functions and their graphs can be an opportunity to *learn how to learn* about this very important idea, and to learn how to use it to think critically about mathematical statements and solve mathematical problems. It can be a key to "belonging" to the community of learners who know they can succeed at math in high school. Or it can lead students to believe that math doesn't make sense and/or they are not smart enough to get it. Whether or not a student comes out of the study of functions having made progress toward the deeper learning competencies depends on how the content is taught. As the following two lessons suggest, teachers can teach functions in very different ways. My purpose is not to compare two particular teachers, but to compare two different kinds of teaching.

Teacher A: Providing a Conventional Introduction to Slope

In the first math classroom, Ms. A stands at the board in front of her class, beginning a lesson that focuses on rate of change in linear functions.[7] The goal of the lesson is projected on a screen at the front of the room:

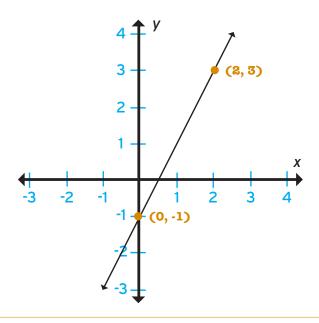SWBAT find the slope of a line given two ordered pairs.[*]

Ms. A tells her class that this is their introduction to "slope," and they will connect finding slope to graphing lines. For a few days, they have been graphing lines using tables of ordered pairs like this one:

x	y
0	-1
2	3
3	5
4	7

They have not yet talked about what it means—or why it matters—if the line slants up or down or goes horizontally across the graph, or if it is "steep" or "gentle".

Before working with the table of ordered pairs, the students have copied this definition of a function from the board into their notebooks: "A function is a relation between a set of inputs and a set of outputs with the property that each input is related to exactly one output." On the same page, they have also written:

input = x
output = y

At the beginning of the lesson, Ms. A draws a horizontal line crossing a vertical line on the board. Standing next to the drawing, she gestures up and down, then left to right, saying, "The graph is divided into four quadrants by two axes: the horizontal x-axis, and the vertical y-axis." She then puts arrowheads at the ends of the lines, and writes an x to the right of the horizontal line and a y at the top of the vertical line. Pointing to their intersection, she says that they cross at a point called the "origin." (The words "quadrant," "horizontal," "vertical," "axis," and "origin" are projected on a screen next to what she refers to as the "graph.") Next, she ticks off equal segments on both lines and puts numbers next to them. She then draws a diagonal line from the top right to the bottom left of the graph, and she darkens and labels two points on that line, resulting in a picture that looks like this:

She then reads from a projection on a screen next to the board: "Remember, every point on the graph can be labeled with two numbers, x and y, and indicated by the ordered pair (x, y)." This is review. Ms. A's students wrote this in their notebooks when they first learned to use tables like the one above to make the corresponding dots on a graph and connect them with a line. The only x and y on the board are the ones labeling the x- and y-axes; the letters are not visually associated with a point on the graph. One of the students asks the boy next to him whether the "y" on top of the vertical line and the "x" next to the horizontal line have anything to do with the (x, y) on the screen. He does not ask the teacher his question, because he knows this is the part of the lesson where the teacher puts things up on the board or screen, and the students copy them in their notebooks. His seatmate writes something on a scrap of paper we cannot see.

Ms. A next flashes what she calls the "definition of slope" on the overhead screen:

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in y}}{\text{change in x}}$$

She tells students to copy this into their notebooks. Thus another "x" and another "y" appear in front of the class, but the teacher does not make connections among the three uses of the same two letters.

She then says, "The slope of a line is usually labeled by the letter m," and flashes another equation:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

She tells students to copy this too, "because this is the rule for how to find slope."

She reminds the class that (x_1, y_1) is one point on a line and (x_2, y_2) is another. She further asserts that it is "really important to keep the x and y coordinates in the same order in both the numerator and the denominator, otherwise you will get the wrong slope." She then tells students that learning to find slope is important so that you can use a "table" to find a "line" and use a line to find an "equation."

As the teacher passes out graph paper, she tells her students to use it to copy the graph she has drawn on the board. She explains that one finds the slope of this line "by calculating the change in y," being careful to "start with the dot on the top." She writes the equation:

$$m = \frac{3 - (-1)}{2 - 0}$$

She asks the class what they get when they subtract 3 – (-1). Some students raise their hands, but she does not call on anyone to give the answer. Instead, she asks a student with his hand up to remind everyone of the rule for subtracting a negative. He says, "Two negatives make a positive." So, she says, the "change" or "difference" in y (the numerator of the fraction) is 4, and she writes a 4 next to 3 – (-1) and puts a line under it. She next asks, "What is the change in x?" Pointing to the board, she answers herself: "To find that, we subtract, 2 – 0." She writes a 2 under the line below the 4, an equals sign next to the 4, the line, and the 2, and another 2 next to the equals sign, and says: "So, the slope of this line is 2." She tells the students to copy what she has written on the board under the graph.

Next, she passes out a sheet with several line graphs similar to the one on the board, each with two points labeled, and tells the students to work independently "to find all of the slopes by following the formula we just learned." She goes around the room while they work, answering questions and putting check marks next to correct answers and x's next to answers that are wrong, asking students to "do the wrong ones over and check in with me again." Before the bell rings, she says, "Remember, this is going to connect with tables and equations, which we will be working on soon."

WHY THIS TEACHING MAY BE CONSIDERED SATISFACTORY

Ms. A's lesson is well organized, and the facts and procedures she presents are mathematically accurate. She introduces two formulas and demonstrates how to plug values into one of them to find the slope of a line. She asks students to write those formulas in their notebooks and copy the work she has done on the board. She gives them a textbook definition of the new academic term "slope." She builds her introduction on terms students have heard before—quadrant, horizontal, vertical, axis, and origin—and

on their representations in graphical form, which students should have learned to make. To some extent, she may be re-teaching these terms and representations for the benefit of students who have never encountered them or have forgotten what they mean—and in this part of the lesson, she is quite animated, speaking, drawing, and showing slides she has prepared, while students appear to be "on task," actively listening, watching, and copying material into notebooks.

WHY THIS TEACHING DOES NOT SUPPORT DEEPER LEARNING

To be successful in this class, students need to demonstrate knowledge of the system the teacher presents. They are learning that "doing mathematics" means following the rules laid down by the teacher, and "knowing mathematics"—and being successful in this subject—means remembering and applying the correct terms and the correct rule when doing an assignment. They are also learning that mathematical truth is determined when their answers are ratified as correct by the teacher or the textbook. The ratio of right answers to wrong ones on their graded papers will signal whether or not they belong among those who are "good at math."

While the teacher provides conventional terms, definitions, and formulas, she leaves out a number of key terms and concepts, including some that could be very helpful to her students. For example, she does not use the phrase "rate of change," even though the formula for finding slope describes the ratio between how much the value of "y" and the value of "x" change from one point to another. Although she tells the students that two points are used to find the slope, she does not mention the fact (a remarkable, even beautiful, bit of mathematics, some would say) that they can *use any two points on the line, no matter how close together or far apart, and the slope will always be the same*. What she has missed here, in other words, are opportunities to pique students' curiosity: *Why does it work? Does* that *always* work?

By the time they arrive at high school, most students in the U.S. have come to believe that they are not cut out for any math more complex than arithmetic (Asante 2004; Sanchez, Zimmerman, and Ye 2004; Lipnevich et al. 2002). They have learned that when their teachers put mathematical formulas up on the board, they are not expected to respond with curiosity and wonderment. Rather, they are expected to listen to the teacher, follow directions, and, perhaps, try to memorize what they write down in preparation for tests (Hiebert et al. 2003). The students know that once the test is over, they will move on to a new topic, and not be likely to see or hear these things again.

Further, in this lesson segment, the teacher communicates that mathematics is a fixed system, in which the student's role is to learn the rules for how to operate within that system. She does this by extending knowledge to students as facts and procedures in a compressed form, briefly unpacking that knowledge with an example and a drawing, but not opening it up to question or interpretation. Students do not practice communicating complex concepts or using mathematical vocabulary, nor do they have the opportunity to use graphs and equations to help them solve problems. They do not have chances to give or receive feedback in their interaction with peers.
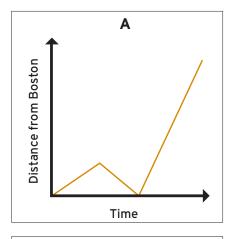
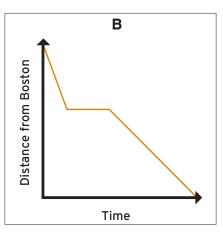## Teacher B: Teaching this Content for Deeper Learning

In the second lesson, we observe the exchanges Ms. B has with her students and the way she structures students' interactions with each other. I offer this lesson as an illustration of the everyday work of deeper teaching, which involves not only pushing students toward the deeper learning competencies, but convincing them to participate in a very different way of "doing school."[8]
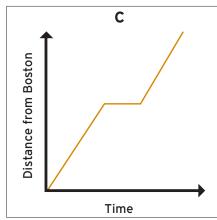
GIVING A DIFFERENT KIND OF TASK, ONE THAT ASKS STUDENTS TO MAKE SENSE OF THE CONTENT

Like Ms. A in our previous example, Ms. B is standing in front of the room when we come in. She is introducing a new activity, which she tells students they will see multiple times, and which, she adds, is connected to "what we have been doing with functions." She begins by taping five pieces of chart paper to the whiteboard, covering the space across the front of the room. She says these are "three coordinate graphs" and "two verbal descriptions" that represent journeys of people traveling by car. She points to the place on graph A where the vertical and horizontal lines meet, and she says that at this point the cars are at Boston, and no time has yet passed.

The papers look like this:

A

Distance from Boston

Time

B

Distance from Boston

Time

C

Distance from Boston

Time

1

Annie was driving fast on her way from New York to Boston when she was stopped by a police officer. He gave her a warning, and she proceeded more slowly back to Boston.

2

Ryan drove from Boston to New York at a constant speed and made a stop for gas along the way.

As she puts up the papers, she explains how the class will proceed, namely by "looking for structure" and "connecting representations." She says, "We are going to look at a graph representation and a verbal description, or real life scenario, in words. So, I'll set up all of these representations here."

From a deeper teaching perspective, it is as important to consider what the teacher is *not* doing when she puts up these charts as it is to understand what she *is* doing: she does not distract students from the task at hand—making sense of the concept of rate of change—by bringing in new symbols or terms, such as "positive slope" and "negative slope"; she does not mention that all of these lines are associated with equations; and she does not write those equations on the board. Rather, she sticks to the interpretation of graphs. In terms of the Common Core Standards for Mathematics, she focuses solely on Algebra Standard HSF.IF.B.4:

> For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: *intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.

Her goal is to build a foundation for understanding the meanings of terms and formulas that students will learn in subsequent lessons. By linking the graphs with verbal descriptions, she enables students to think about what these abstract representations have to do with something familiar. The familiar will provide an anchor as they move to manipulating abstract symbols, helping them judge whether what they are doing "makes sense."

In each of the graphs and scenarios, the distance from Boston is related to how much time has passed since the beginning of the trip. (Formally, we might say that the distance from Boston is a *function* of time. The *rate* at which the distance *changes* in relation to time determines the steepness of the lines.) The one thing she is asking

students to investigate right now is why the lines are sometimes flat, sometimes steep, sometimes slanting upwards, and sometimes slanting downwards. She is, in short, giving her students an opportunity to engage with the visual representations and figure out what they mean in terms of real-life scenarios (Arcavi 2003).

Familiar scenarios like this often help to engage students in classroom activities, but evidence suggests that they are also key to the development of efficient mental structures that provide interpretive perspectives on problems and how they might be solved. In other words, scenarios are tools that can carry knowledge from one domain to another (Bransford, Brown, & Cocking 2000). Students need to be able to do this in order to learn deeply in secondary school, and they need to *know they can do it*, so that they come to feel secure about their ability to move into unfamiliar academic and professional territory. Ms. B teaches in a school whose students are 50 percent Hispanic and 45 percent African American. 85 percent are classified as "low income" and 96 percent as "high needs" (MA Department of Education 2015). Students in these categories are less likely to finish high school, less likely to attend college, and even less likely to graduate once they get there (U.S. Department of Education 2012, 2015). They thus have the most to gain from acquiring the belief that they can make sense of mathematics and succeed at learning it.

BUILDING ON WHAT STUDENTS KNOW, NOT FAULTING THEM FOR WHAT THEY DON'T

As she waves toward the pieces of chart paper, Ms. B asks if anyone remembers any of the different ways of showing a function that they have already studied.[9] The first student to answer says, "y-intercept." She is in the right ballpark, for the y-intercept *is* an important part of each graph, as it shows where the car started out in relation to Boston. But it does not "show" a function. In order to show a function, something would need to show a *relationship* between two quantities that vary in relation to one another.

By asking students to tap into their prior knowledge of functions, Ms. B opens herself up to the risk that students might not be able to answer her correctly. The question is, how does she make them feel okay about volunteering ideas that may not be right? In this case, she has to respond to an answer that is "in the ballpark" in a way that lets the student know she is on the right track, while also taking her and the class further down that track toward mathematical precision.

Ms. B responds, "We've looked at the y-intercept. And we can see the y-intercept on all of them, in all these representations." She then gestures at the coordinate graphs. She chooses to recognize the relevance of the student's contribution in a way that will keep her engaged, while drawing her attention, and the attention of the class, to the different ways of showing functions and how to connect them with one another. She also pulls the whole class in by repeatedly using the pronoun "we"—"*We've* looked at the y-intercept. And *we* can see . . ." Her language communicates that engaging with the content is a group effort, and that this content is not something "out there" and impersonal, but something "we," in this room, are working on. Ms. B's use of "we" is a deliberate choice, like many of the words she uses to indicate that the class is involved in a different kind of learning than what happens in Ms. A's classroom. It indicates that she and the students are co-constructing ideas together, working collaboratively on building and maintaining a shared sense of the mathematical concepts under study.[10]

By making a habit of responding in this way to incorrect answers, she shows her students that she can be trusted not to belittle or embarrass them, and that learning is a process of connecting what you know to what you need to learn. This kind of teaching move encourages more student contributions, makes connections between this and previous lessons, and builds students' intellectual courage to speak in front of the class, even if their thinking needs to be revised at some point. By taking the emphasis off right

» She and the students are co-constructing ideas together, working collaboratively on building and maintaining a shared sense of the mathematical concepts under study.

and wrong answers, she is constructing a classroom culture in which mistakes are an opportunity for learning, rather than a situation in which teachers judge students, and students judge themselves, to be "not good at math."

Ms. B is using what students know to move on to new material, a well-known and powerful strategy for building commitment to learning (Fennema & Carpenter 1996). This move is one of many that make it possible for her students to try on an identity that includes being good at math (Boaler & Greeno 2000). By using a student contribution to call attention to something important in the lesson—even though the comment does not answer the question she had posed—she is positioning every student who answers a question as someone who belongs in a community of learners. Or, to phrase it in terms of deeper learning, she is building "academic mindset" by challenging widely held beliefs that some people are just "good at math" and the rest can't learn it (Dweck 2007). Every time she makes a move like this, she is attempting to connect students' sense of who they are and who they want to become with learning as a social and worthwhile activity. She is teaching the student who answered and all of her classmates to be learners.

USING TOOLS AND ROUTINES THAT SUPPORT COLLABORATION AND COMMUNICATION

After soliciting additional answers, such as "graphs" and "table," to the question of how they have studied functions, Ms. B says that the goal of this activity is to "make connections between two things that look nothing alike but have the same underlying structure"—that is, to figure out which of the written scenarios correspond to which of the graphs. But, she adds, she expects them to do more than just figure out which graphs match up with which narratives; she wants them to practice skills they have worked on before, "like looking for structure in numbers and thinking like mathematicians."[11] She adds:

> It's going to be really important that you are *justifying* your representation and using language to connect both representations. So, for example, you could say, "The graph shows," and you name something that you see on the graph, and you say, "It's shown in the story by," and then you name something that you're seeing in the story.

She also explains that they will be using a classroom technique that they will repeat several times over the coming weeks: first, they will think about the problem alone, and then they will share their thinking with a classmate and then with the whole class.[12] This activity structure is designed to require every student to collaborate and communicate about a clearly defined task, working in pairs to hash out their answers and decide how to explain their reasoning to the rest of their peers.

This kind of talk—openly discussing their assumptions, making sure they all mean the same things by the words they're using, and being explicit about their thinking—is unfamiliar to Ms. B's students, almost to the point of learning a new language.[13] One only learns to collaborate and communicate by trying to do these things and being coached to improve.[14]

What Ms. B is trying to get these students to do is radically different from what typical American students are used to. Many high school students simply do not talk in class because they don't want to call attention to themselves in an academic setting. They prefer the anonymity of worksheets and lectures, and for the most part their teachers are willing to oblige (Powell, Farrar, & Cohen 1985). But Ms. B is asking her students to justify their answers to their peers and challenge each other's justifications until they arrive at a solution that is mathematically legitimate (Staples, Bartlo, & Thanheiser 2012; Blanton & Kaput 2005; Kaput 1999). Further, she tells them explicitly that this is her goal:

> I'm going to be pushing you to be connecting back and forth between the two [stories and graphs] and really using evidence to convince your partner, and then the whole class. By the end, you're going to have to create your own representation for a graph, and you're going to write your own real world description.

This is Ms. B's equivalent of the SWBAT in Ms. A's classroom: she gives her students an end-of-class goal that will require them to invent something new, applying what they have learned from the earlier phases of the activity. While she provides many supports, she leaves the fundamental intellectual work of connecting the stories with the graphs to the students. They are to *listen* to how their partner matches the representations, ask clarifying questions when necessary (Chapin & O'Connor 2007; Smith & Stein 2011), agree on a conclusion, and come up with a way to explain it to the class.

MAKING IT SAFE AND PRODUCTIVE FOR STUDENTS TO PUBLICLY PERFORM THEIR ACADEMIC COMPETENCY

When the allotted time for independent pair-work ends, Ms. B calls the class together and asks for "one match." Tatiana[15] raises her hand, and the teacher addresses her not as an individual but as "you and Floriana," the pair who worked together on the task. By attributing the finding to a pair, she communicates that collaboration is the norm. She also lowers the personal risks Tatiana might face if she were asked to speak in front of the class only on her own behalf. Having thus put Tatiana at ease, Ms. B asks her to speak loudly, expecting that everyone is listening and anyone might have something to say about her answer:

> Ms. B: First match? Tatiana, can you share what you and Floriana found? Nice and loud.
>
> T: We found that B and 1 go together because we said that she was going a certain speed. And then the police stopped her. And then if it was going to, from Boston to, to New York, it would have started there [points to the lower left hand corner of the graph], but it didn't start there. So it started from the top. Which means that she started from New York.
>
> Ms. B: Great, so can I have you two come up to the front? Okay, now Floriana, you're going to stand next to the representation, so make sure you can see both. And you are going to point to what Tatiana is saying. So Tatiana, can you turn and face your class? Nice and loud, and Floriana is going to help you by pointing to what you are saying, okay? Go ahead. Explain how you found your match.

Repeating what she said before she came up to the board, Tatiana addresses the class:

> T: We found that B and 1 go together because we said that she was going a certain speed and then the police stopped her, and then, if it was going from Boston to New York, it would have started there [pointing to the origin], but it didn't start there. So it started from the top which means that...

As she speaks, her partner Floriana points to the section of the graph that represents the time before the police stop, then to the section that shows the stop itself, and then to the place where the line intersects the vertical axis, "at the top," showing that the car did not start in Boston (at the origin).

In this activity, the teacher has created a structure for public collaboration, giving each student in the pair a role—and explaining those roles publicly, so that everyone in the class can begin to learn them—as well as a way to take ownership of their shared assertion that graph B connects to story 1.

She doesn't begin by asking which match they found (which might suggest that their job is simply to give an answer, right or wrong). Instead she presses for an explanation of "how you found your match." This puts the girls in a "think on your feet" mode that could be stressful for many students. Ms. B knows, however, from having listened to the pair's conversations, that these two girls have come to a clear, shared understanding of the material. Further, she asks them to face the class and speak loudly, communicating that their audience is everyone in the room, not just her. By asking them to explain their thinking in front of their peers, she gives them a relatively safe opportunity to show that they are capable of complex mathematical reasoning and, in a larger sense, to try on the identity of "serious math student" (Lampert 2001). This in turn shows their classmates that they too can likely do such work.
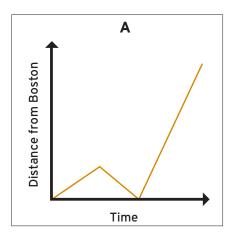
Tatiana begins the pair's justification that graph B goes with story 1 by focusing on the point that shows the speed of the car when it is stopped by the police, after which a horizontal line shows that the car's distance from New York and Boston did not change for a period of time. She clinches their argument with a logical move called a "counterfactual," raising the possibility that the car "was going from Boston to New York" and concluding that this would lead to a different graph. Focusing on the beginning of the journey, she explains that the car could not have started in Boston because *the line* that represents the journey does not start at the origin. Since this is the only one of the three graphs whose line does not start at the origin—it begins "at the top" and goes diagonally downward—it must be the one that goes with story B. That is, the girls focused on the y-axis, which measures "distance from Boston,", and they noticed that the driver ended there.

Communicating in this way is not a natural activity for either teachers or students. Rather, it is a deliberately constructed instructional design intended to engage students in challenging-but-doable tasks that reinforce the idea that they are capable of deeper learning. Ms. B's choice to call on Tatiana was also purposeful. Ms. B had listened to what the two girls said to one another as she circulated around the classroom. They seemed secure in

their understanding of why B and 1 should be connected, so she knew they would likely be able to take the next step of presenting their thinking to the class.

STRUCTURING STUDENTS' PRODUCTIVE STRUGGLE WITH CORE CONTENT

In the third part of the lesson, Ms. B deliberately takes students into new territory. She assigns pairs of students to write the story that goes with this graph:

Coming up with a matching story for this graph entails a deliberate challenge. The two other graphs include a horizontal line between two upwardly sloping lines, signifying a stop in the journey. But in graph A, there is a downwardly sloping diagonal between two upwardly sloping diagonals. The questions are: What happens in a functional relationship when the graph suddenly switches from "going up" to "going down"? And what does it mean that the second line segment goes down to the x-axis and the next segment goes upward, more steeply than before?

This segment of the graph represents an important mathematical idea that is fundamental to linking graphs with stories, but tends to be harder to grasp than the narrative interpretation of the upwardly sloping and horizontal lines because it involves a visual contradiction that can only be resolved with mathematical reasoning. A downwardly slanting line might seem to suggest that the car is slowing *down*, and when the graph turns upward again, it might seem to suggest that the car is speeding *up*. But this is a common misinterpretation of graphs that represent time, speed, and distance (Stevens & Hall 1998). In fact, the second, downward segment shows that the car returns (at the same speed) to Boston, and the third segment shows that it sets out again to New York, moving

at a faster speed (perhaps to make up time for having had to go back to Boston).

In formal mathematical terms, we would say that the first segment has a positive slope, and the second has a negative slope, while the third has a greater slope than the first. For someone who does not regularly work with such graphs, though, the visual and verbal cues can easily overtake the conceptual.

In this beginning algebra class, then, Ms. B asks students to use what they know to confront a mathematically interesting inconsistency that leads them to need to come up with a new idea. Once they grasp one mathematical concept, she challenges them with a problem that requires them to figure out another more sophisticated concept.

This is a fundamentally different way to learn math—or, in a larger sense, to be a student—from what we saw in Ms. A's classroom, where the typical assignment required students to use a memorized formula provided by the teacher to figure out pre-established answers to the teacher's questions (Cohen 1988, 2011; Cohen & Spillane 1992). A New York City high school teacher, Constance Bowen, described this sort of "deeper" instruction:

> These are rich questions that elicit reasoning and build understanding about how the variables being measured relate. If we had simply asked them, 'What is the slope from point A to point B,' then they would have applied the algorithm without any thought of its meaning... Students typically grab onto the formula as the method and actually don't build understanding or the ability to describe what is happening in the relationship between the two variables at play.

How Deeper Teaching is Distinct

The aim of both these lessons is for students to learn something about the mathematical concept of "slope." Most American classrooms resemble the first teacher's: teachers act as judges, referring to their own conventional content understanding to decide what is right and wrong. Such teaching does not enable students to recognize that they—not their teachers or textbooks—are responsible for their learning (Boaler & Greeno 2000). Moreover, in conventional classrooms, students experience only a small part of the rich content of mathematics. Sticking with straightforward questions and known answers, they come into contact only with what Alfred North Whitehead, in his 1929 treatise on

"The Aims of Education," called "inert ideas...ideas that are merely received into the mind without being utilised, or tested, or thrown into fresh combinations" (p. 1). Whitehead claimed that by passively receiving such inert ideas, learners acquire information that they can express but not use.

Whitehead's advice for improving education was very much in line with the deeper learning proficiencies, focusing on core content, active engagement, and attitudes that increase productive learning habits:

> Let the main ideas which are introduced into a child's education be few and important, and let them be thrown into every combination possible. The child should make them his own, and should understand their application here and now in the circumstances of his actual life. (p. 2)

Current research on learning bolsters Whitehead's argument. [16] In 1999, the National Research Council's Committee on Developments in the Science of Learning published *How People Learn: Brain, Mind, Experience,* *and School*, an exhaustive review of extant research. Briefly put, that work described three main principles (National Research Council 2005) for enabling students to learn successfully and deeply: teachers must build new understanding on the relevant knowledge and experience that students bring to the classroom; they must help students to integrate factual knowledge into a network of concepts to support knowledge use in new situations; and they must support students' capacity to become aware of and engaged in their own learning and ability to decide whether their answers make sense (see also Bereiter & Scardamalia 1985).

Teaching according to Whitehead's proposals and the NRC learning principles is congruent with what I have called deeper teaching. To do this kind of teaching, teachers need to make appropriate decisions about what to teach, build on students' current understandings, use methods of instruction that link the two together, and respond to each student's contributions with feedback that is carefully formulated to communicate to all students that they are capable of becoming competent.

Table 1. Different Ways of Thinking about Knowledge can be Combined with Different Ways of Thinking about Teaching		
	Teacher explores student knowledge indirectly	**Teacher explores student knowledge directly**
Teacher views knowledge as fixed and searches for congruence between student knowledge and fixed knowledge	1. [The teacher] uses multiple-choice tests, homework or seatwork handouts, and similar devices to probe student knowledge. *Teachers require little interactive skill and little knowledge of exploratory techniques.*	2. [The teacher] uses simple question and answer or formal recitations to probe students' knowledge. *Teachers need some interactive skill and knowledge and some capacity to frame appropriate questions and quickly assess answers.*
Teacher views knowledge as the outcome of inquiry and searches for signs of minds at work	3. [The teacher] uses essays, journal writing, and the like to explore what students know and how well they can explain it... *Teachers require little interactive skill and knowledge, but they need considerable specialized knowledge of the subject to ask good questions and respond thoughtfully to students' answers.*	4. [The teacher] uses discussions, debates, extended colloquies, and other direct discourses to probe what students know. . . *Teachers require considerable knowledge of the material, interactive skill, and ways to combine the two.*

To illustrate how this sort of teaching differs from other kinds of instruction, Cohen (2011) offers a simple table that shows how different ways of thinking about knowledge can be combined with different ways of thinking about teaching (Figure 1). Deeper teaching falls in cell 4 of the table. It requires teachers to have very different knowledge and skills than what they need for the kinds of teaching in cells 1, 2, and 3.

To show further how deeper teaching differs from conventional instruction (in the terms of the table: how teachers define what it means to "know something" and whether or not teachers interact directly with students' thinking) consider the following contrast between the teaching of Ms. A and Ms. B.

Midway through her lesson, Ms. A told her class, "The slope of a line is usually labeled by the letter m." On the overhead screen, she flashed the equation:
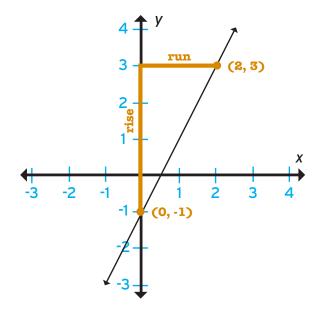
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

She told her students, "This is the rule for how to find slope." Referring to a graph she had drawn, she said that one finds the slope of the line "by calculating the change in y," being careful to "start with the dot on the top." She then wrote the equation:

$$m = \frac{3 - (-1)}{2 - 0}$$

If Ms. A had asked her *students* to make the equation for finding the slope of the line on the graph, rather than doing it herself, they might have had an opportunity to practice what Whitehead called "throwing inert ideas into fresh combinations," edging toward what the deeper learning proficiencies refer to as "solving problems and thinking critically."

But Ms. A did not even make it explicit that she arrived at this equation by inserting the numbers associated with the points on her graph. Nor did she go back to the graph to

explicitly represent the link between these subtractions and the visual "rise" over "run" in what she called "the definition of slope," which would have entailed adding lines to the graph that looked like this:

Linking the equation to the rise and run lines would have given the students an anchor not only for understanding what the equation for slope represents, but also for seeing that **3 − (-1)** is the length of the "rise," that is, the line from 3 to -1 on the y axis. Ms. A might have asked her students to use the graph to talk about *why* 3 − (-1) is 4 not 2, relating the quantity 4 to the length of the "rise." She might even have linked what looks like a fraction to the idea that slope is a *ratio* between the change in x and the change in y, which would stay the same for any two points on the line. This might have at least given students an exposure to the conceptual framework that leads to core academic content or understanding "key principles and relationships within a content area and organiz[ing] information in a conceptual framework" (Hewlett Foundation 2013).

Instead, when Ms. A asked the class what they got when they subtracted **3 − (-1)**, she did not wait for an answer but immediately asked a particular student, presumably one she knew she could rely on, to remind everyone of the rule for subtracting a negative. Repeating the rule he had memorized, he expressed an inert idea, as Whitehead calls it. Then Ms. A did the calculation herself, unwilling to risk that a student might call out the wrong answer. No student

even had a chance to apply the rule. To the extent that we might call what Ms. A did when she inserted the values from the points on the graph into the equation for slope "intellectual work," she did that work, not her students.

The question Ms. B asked to elicit students' memories of the different ways of showing a function and the way she responded to their answers offers a signal contrast to Ms. A. Recall what Ms. B did when she asked a question that was designed to elicit students' memories of the different ways of showing a function that they had already studied. The first student to speak said "y-intercept," which is not a way of showing a function. But Ms. B did not judge the response "y-intercept" right or wrong, which would have positioned her as the "owner" of the mathematics. Instead, she responded in a way that connected what the student said with what they would be working on. She also positioned students to try out being mathematical thinkers. By making a habit of not judging their answers immediately, but giving them a chance to make sense of things themselves, she let them know that the connections they make are valued, and

that her classroom is a safe place to try on the "academic mindset" one needs to do mathematics.

As Ms. B's work illustrates, deeper teaching requires setting up interactive structures for students that deliberately reposition them in relation to academic content and to one another, incorporating their agency, interests, and knowledge in ways that make accessible the meaning and relevance of academic material. Perhaps the most profound segment of Ms. B's lesson is the part where she moves to the graph that does not have a story, and asks the students to write the story. Putting too much emphasis on the familiar visual and not enough on the unfamiliar symbolic is a problem that comes up over and over again in algebra, geometry, and calculus, where graphs that model abstract mathematical relationships are often interpreted as pictures showing a concrete phenomenon (Leinhardt, Zaslavsky, & Stein 2000). Ms. B knew that if she gave her students two sets of graphs and stories to link together, and then a third anomalous graph, she would be building a foundation for one of the more important aspects of mathematical understanding.

CONCLUSION: TOWARD DEEPER TEACHING

I began this paper by asking how teaching would have to be different if learning were to be deeper. Drawing on the learning principles derived by the National Research Council, and comparing the deeper teaching of Ms. B with the more common and familiar teaching of Ms. A, we might say that:

1. Ms. B needed to be able to *elicit students' current understandings and build from them* toward the deeper learning competencies. In order to do this, she made student reasoning–rather than proffering known answers to teacher questions–the basis of teacher-student and student-student interactions.

2. Ms. B needed to know her subject well enough to *decide to teach* and *be able to teach* core content, critical thinking, problem solving, collaboration, communication, a disposition to learn, and academic mindset. This involved choosing rich tasks, developing academic language for describing and communicating about ideas, and structuring interaction so that it was safe for students to express partially formed thinking in the classroom.

3. Ms. B needed to *choose* and be *able to use* methods of instruction that link what the teacher is teaching with students' current understandings. Students need structured, predictable routines for surfacing their interpretations and conceptions, and Ms. B needed to use strategies for linking individual work and small group work with the learning agenda for the whole class.

This is a tall order.[17] For this reason, such teaching is sometimes referred to as "ambitious teaching" (Lampert, Boerst, & Graziani 2011). While #2 has to do with the skills and dispositions that teachers need to acquire ahead of a particular instance of teaching, much of the work entailed in #1 and #3 requires the teacher to make quick judgments while working face-to-face with multiple students. The knowledge underlying those judgments includes the core ideas of a subject, how they relate to one another, and why they are important, as well as how students are likely to think about the content of a lesson, in order to prepare to connect their prior knowledge with the core ideas being taught. The question, then, is: How can this level of complexity be managed in day-to-day teaching?

The way Ms. B interacted with students and mathematics did not just "come naturally" to her, although she entered teaching with a strong mathematical background. Nor did she invented it single-handedly. Although Ms. B is a beginning teacher, she could enact the lesson because she had learned a set of routines for carrying out the kinds of complicated interactions that support deeper learning. She did not learn these routines in a context-free, content-neutral way, for researchers have found that it is difficult for teachers to use teaching routines taught in the abstract in particular settings (Loucks-Horsley & Matsumoto 1999; Ball & Cohen 1999; Cohen & Hill 2001). Rather, she learned them in repeated cycles of observing, planning, teaching, and analyzing an Instructional Activity (IA) called "Connecting Representations."

> » She made student reasoning—rather than proffering known answers to teacher questions—the basis of teacher-student and student-student interactions.

This IA is one of a small number of protocols that can be used to teach mathematics ambitiously. IAs have been used for many years to organize balanced literacy instruction,[18] and they are now being designed for use in mathematics, science, and social studies.[19] In effect, they are templates for organizing classroom instruction in a way that makes room for students to problem solve, communicate, and collaborate. They provide students with support for learning to learn and developing academic mindsets.

Making the structure of an IA like Connecting Representations a regular part of lessons reduces the cognitive load of ambitious teaching, so that teachers can pay close attention to students and connect what they are doing to elements of the domain under study, thereby positioning them as agents of their own learning. IAs are not one-off tasks that a teacher might find in a resource book or on the internet; rather they are protocols that are meant to be used repeatedly with varying content. They are deliberately designed to accomplish ambitious aims by specifying who (teacher and students) should be doing what, with whom, when, and for how long.

Instructional Activities also provide very specific guidance as to how teachers can respond productively to students' contributions in class. Their design limits the conceptual territory into which students might venture, so the teacher can think through, in advance, what kinds of questions and points of confusion are likely to arise. Further, action protocols for each activity are the same across grade levels and levels of teacher experience. Whenever teachers and students do an Instructional Activity, they are repeating patterns of social and intellectual interaction that students must learn in order to accomplish deeper learning goals.

As McDonald and her colleagues (2013) put it in a recent article:

> Instructional activities are episodes that have beginnings, middles, and ends, and within those episodes they clearly guide how teachers and students are expected to interact, how materials are to be used, and how classroom space is to be arranged. The reason for this detailed specification is to create a container within which a novice might rehearse the relational and improvisational work that teaching requires. Well-crafted instructional activities can also allow teachers to attend to how children's ideas are given voice in the classroom, and how participation structures in the classroom position students competently and enable children to

orient to one another's ideas and meaningful ideas in the content. They also challenge teachers' ideas about who can learn and what it means to learn in school (McDonald, Kazemi, & Kavanagh 2013).

A system of Instructional Activities can organize teaching and learning to regularly include: individual think time for students; students explaining their thinking to one another; making student thinking public by representing it for the class; and connecting student reasoning to core mathematical content.[20] Repeatedly using the practices that support these kinds of activities turns important elements of academic engagement into habits. Through repetition, both teacher and students acquire new intellectual and social skills and dispositions. More importantly, perhaps, both teacher and students acquire new ways of thinking about what it means to teach and learn, and what they are able to accomplish.[21]

How can Deeper Teaching Happen more Broadly?

The question inevitably asked about any ambitious instructional reform is whether it can improve the quality of teaching "at scale." Reformers and researchers concur that extending improvements beyond a single classroom, school, or district is a complicated matter. Researchers find that large scale change can be initiated and sustained when educational resources are coordinated systematically. The list of resources required for such efforts is relatively long but consistent: curriculum materials; instructional guidance tools, including standards and instructional routines; assessment and record-keeping instruments for teachers and students; common space and time for teachers to learn to use and adapt resources; content-focused instructional leadership; and district support for school-level capacity building (Lampert, Boerst, & Graziani 2011; Cobb & Jackson in press; Bryk et al. 2010). The operant word for linking these resources is *coherence*—if all of the tools available to improve instruction are not aligned in use, any one tool, by itself, is unlikely to improve instruction (Cohen, Raudebush, & Ball 2003).

Ms. B is part of a coherent, albeit small, system of instructional improvement. She works in a group of teachers who use the same set of Instructional Activities, which are aligned with the Boston Public Schools' academic goals and targets and the Massachusetts Common Framework, which are, in turn, aligned with one another and the Common Core State Standards. The students of

Ms. B and her colleagues are assessed using the PARCC, which aligns with the city, state, and Common Core. Further, the teaching practices in the IAs are those advocated by the district's Department of Instructional Research and Development, while the math content is drawn from the district's Scope and Sequence for the appropriate grade level, which is aligned with the PARCC assessment schedule.

The work on Instructional Activities that Ms. B and her colleagues do together is led by senior teachers and teacher educators who believe that students are capable of acquiring the competencies identified as deeper learning. Ms. B and others in her group rehearse in front of one another, and coaches help them hone their skills in implementing the routine parts of the activities. Since IAs are designed to elicit students' mathematical input, this group of teachers prepares to teach with an IA by deliberating together about appropriate ways to use student input to achieve learning goals, pooling their knowledge of students and mathematics. They also collectively plan adaptations to the IAs that suit their students and the mathematics they plan to teach. On occasion, they watch one another's lessons and give feedback on the practices they are learning together.

Such collectives can be built when teaching is supported by common instructional and assessment tools and opportunities to learn to use them. As individual teachers use common resources in instruction, they can make ambitious teaching happen across classrooms by scaffolding the risky and complex work of engaging each student in learning to perform authentic tasks. And because individual teachers use *common* resources, they have the shared language and artifacts that are pivotal for working together on common planning and on the shared evaluation of lessons and students. That is, they share a framework within which to figure out how to use what they know about mathematics and the students they teach.

Similar systematic instructional approaches to making teaching and learning "deeper" are occurring in several places around the country. For example, they can be seen in Balanced Literacy programs in the Chicago Public Schools; whole-school interventions like America's Choice and Accelerated Schools; intellectually ambitious systems of public charter schools like Achievement First and Aspire; schools participating in the Big History Project, the Literacy Collaborative, and the Reading and Writing Project; and public and private Montessori schools (Bryk et al. 2010; Cohen et al. 2013; Rosenberg 2012; Colby & Wikoff

2006; Cossentino 2005). None of these, except perhaps Montessori and the charter schools, have aspirations as comprehensive as those expressed in the deeper learning competencies. Nonetheless, we can learn from how they have operated.

All of these systems disrupt the conventional relationships among teacher, students, and content with deliberate, practice-sensitive designs for instruction. Each design is based on answers to a set of fundamental and closely related questions: First, what do we think students need to learn? Then, what do we know or believe about how those things are learned? Finally, how should the classroom be organized to make learning possible—what should students be doing with the content to be learned, and what should teachers be doing to make what students need to do possible?

In all of these cases, in order to build capacity for successful instruction, researchers concur that three aligned developments must happen simultaneously:

> Designing **tools for instructional guidance**, including protocols for enacting named and commonly recognized/shared teaching practices in lesson structures and assessments that give teachers and students feedback about whether their goals are being accomplished;

> **Organizing schools** to be coherent **systems** that support (and do not interfere with) teachers and students in **using those tools**; and

> Building the **individual knowledge** teachers need to use teaching tools and adapt them to particular students with good judgment.

I have addressed the first of these developments above. How individuals learn to use and adapt instructional guidance tools is beyond the scope of this paper, but has been addressed elsewhere (Ghousseini, Beasley, & Lord 2015; Lampert et al. 2013).

Other papers in this series address school organization, but I have focused here on understanding the work of deeper teaching, while also acknowledging that such teaching can flourish only in a school organization that supports it (Elmore 1996; Coburn 2003; Jackson et al. 2015). Using common tools like Instructional Activities, teachers can learn to provide consistent instruction by preparing and planning together, teaching in a way that makes it possible for others to observe and coach them, and collecting and

analyzing records of practice as a basis for refining their lessons and units (William 2007; Kazemi 2008; Stigler & Thompson 2009). Schools need to be structured to support collective teacher learning as a series of improvement cycles that quickly move in and out of practice (Cohen et al. 2013; Stein & Smith 2011).

However, although common tools and well-organized schools are necessary, they are not sufficient. In order to come alive, they require a corps of teachers who have the knowledge of content, students, and the context in which they are teaching to make informed judgments about how to use an instructional design appropriately and how to respond productively to each particular student contribution—year by year, unit by unit, lesson by lesson, and moment by moment. Only in the particular interactions between a teacher and a class can an instructional design be implemented in a way that utilizes its power to achieve the learning goals its designers embrace.

Why Deeper Teaching is Important

In the main, teaching and learning in U.S. schools has not been organized to engage students as agents of their own learning. Neither has it enabled students to break out of their beliefs that some people are "good at" school and others are not, and there is nothing that they can do about it (Dweck 2007). As the National Mathematics Advisory Panel (2008) to the U.S. Department of Education observed:

> Research demonstrating that beliefs about effort matter and that these beliefs can be changed is critical. Much of the public's resignation about mathematics education

(together with the common tendencies to dismiss weak achievement and to give up early) seems rooted in the idea that success in mathematics is largely a matter of inherent talent, not effort (p. 32).

Changing these beliefs and tendencies will not be a simple matter of removing some topics from the curriculum and replacing them with others that are more complex and challenging. It will mean organizing teaching and teacher learning to connect students' sense of who they are and who they want to become with what they learn in the classroom.

I conclude by returning to the idea that deeper teaching is a set of practices that support students in building a new scholarly identity—one that enables them not only to master core academic content, think critically, and solve problems, but also to communicate and collaborate with intellectual confidence and become active agents in their own learning. Indeed, such teaching involves improving students' minds, souls, and habits, as well as their skills and knowledge. It is important to note that these changes are aimed toward students' capacity to succeed in future school and work settings, and that the virtues toward which deeper teaching aims are intellectual and civic virtues.

The distinguished mathematician George Polya (1954) once articulated what he called the "intellectual virtues" needed to do science. Today, we see a call for such virtues in the standards we are setting for what students need to learn (and be) in all academic domains. Polya wrote:

> In our personal life we often cling to illusions. That is, we do not dare to examine certain beliefs which could be easily contradicted by experience, because

>> Deeper teaching is a set of practices that support students in building a new scholarly identity—one that enables them not only to master core academic content, think critically, and solve problems, but also to communicate and collaborate with intellectual confidence and become active agents in their own learning.

we are upsetting the emotional balance. There may be circumstances in which it is not unwise to cling to illusions, but in science, we need a very different attitude, the *inductive attitude*... It requires a ready descent from the highest generalizations to the most concrete observations. It requires saying "maybe" and "perhaps" in a thousand different shades. It requires many other things, especially the following three:

> **Intellectual Courage:** we should be ready to revise any one of our beliefs
>
> **Intellectual Honesty:** we should change a belief when there is good reason to change it
>
> **Wise Restraint:** we should not change a belief wantonly, without some good reason, without serious examination (p. 7-8).

To be deeper learners, students will need to *change their beliefs* about what academic work entails and about their ability to do that work. Given what we know about adolescents, such a change is likely to "upset the emotional balance." Given what we know about educational institutions, it's clear that few schools are prepared either to throw students off balance in this way or to help them regain a firmer footing.

In highly tracked schools, for example, it is very hard to challenge the assumption that some students just can't learn. When test scores deem students "below basic," it is difficult to help those students redefine themselves as competent learners. Teachers like Ms. B not only have to push back on students' prior classroom experiences, but also must challenge the messages about learning and competence that stratified school systems broadcast every day. Deeper teaching is much more than what an individual teacher does in an individual lesson, but everything he or she does in that lesson is essential to supporting deeper learning.

ENDNOTES

[1] Students move toward an identity that psychologist James Greeno calls "intellective." (See Boaler & Greeno 2000.)

[2] See also the *High School Survey of Student Engagement*. Consistently, each year from 2006 to 2009, 65 percent of students reported being bored everyday and only 2 percent reported never being bored. Also consider 2014 dropout statistics, which take us into the inequities in students developing "academic identities." Graduation is a low bar but suggestive of how hard it would be to achieve at a higher level (U.S. Department of Education 2012).

[3] Philip Jackson, Willard Waller, and Dan Lortie were early identifiers of these patterns in the practice of teaching (Jackson 1968; Waller 1932; Lortie 1975).

[4] In the Common Core State Standards for Mathematics (2010), what students need to learn about rate of change and slope are connected thus: "*Construct a function to model* a linear relationship between two quantities. Determine the rate of change and initial value of the function *from a description of a relationship* or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values." (8.F.B.4) They are also expected to "Use similar triangles to *explain why* the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; *derive* the equation y = mx for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*." (8.EE.B.6; emphases mine.)

[5] E.g., option traders study the relationship between the rate of change in the price of an option relative to a small change in the price of the underlying asset, known as an options delta; mapmakers decide on scale depending on what they want to highlight for the user; and coaches train long distance runners based on knowledge of when it makes sense to slow down or speed up in a race.

[6] On this kind of graph, there is a unique pair of numbers associated with every point on an infinitely large flat surface (its *coordinates*). The coordinates describe where that point is in relation to a reference point called the *origin*: How far left or right of the origin is the point? How far above or below? Invented in the 17th century by Descartes, this representation caused a revolutionary leap in the growth of mathematics because it made possible a link between the two separate fields of algebra and geometry. It allowed an easy visual comparison between functions. It played a crucial role in the invention of calculus.

[7] Ms. A is an archetype, not a real person. What she does in this lesson is a composite of the many lessons of this sort I have observed in high schools in the last five years.

[8] Ms. B is a real person. The lesson we see was planned by two Residents in the Boston Teacher Residency Program (Clarissa Gore and Meaghan Provencher) and taught by one of them (Meaghan).

[9] The ways that functions are represented (graphs, verbal descriptions, tables, and equations) and how the elements of these representations are connected is at the heart of the "core" mathematics in this domain. (See Leinhardt et al. 1990.)

[10] Staples (2007) identifies such co-construction as an essential learning practice in secondary mathematics classrooms that seek to promote students' mathematical understanding and engagement.

[11] "Look for and make use of structure" is one of the eight mathematical practices that the Common Core State Standards require teaching throughout grades K-12. But it has long been identified as a key to doing mathematics of all sorts. See for example, Kline (1972).

[12] The design Ms. B is enacting is based on a protocol created by Grace Kelemanik and Amy Lucenta for the Boston Teacher Residency.

[13] They are learning to communicate in what has been called "the mathematics register" by Halliday (1978). He referred to "the discipline-specific use of language employed in mathematics education" (Jablonka 2013, p. 51) as "the mathematics register." It should be noted that

this does not solely refer to specific vocabulary, but also to meanings, styles, and modes of argument.

[14] Individual development toward these competencies cannot be understood without reference to the social context within which they are embedded. See, for example, Wertsch (1988, 1985), Rogoff (1990), and Sfard (2008).

[15] Student names are pseudonyms.

[16] See the "implications for teaching" chapters in the NRC's *How People Learn: How Students Learn: History, Mathematics, and Science in the Classroom* (National Research Council 2004). For additional summaries of the implications of learning research intended to influence the design of teaching, see National Council of Teachers of Mathematics (2014) and Swan (2005).

[17] This short summary of the knowledge, skill, and commitments needed for deeper teaching is similar in content to the list of Mathematics Teaching Practices issued by the National Council of Teachers of Mathematics (2014) to align instruction with the learning goals in the Common Core State Standards (*Principles to Actions*) and the Design Principles for Instruction underlying the lesson guides issued by the Mathematics Resource Assessment project as part of the Math Design Collaborative initiated by the Bill and Melinda Gates Foundation.

[18] For example, see Fountas & Pinnell (1996).

[19] Some of the IAs in use across subjects and grade levels are collected on a website called Teacher Education by Design. They can be viewed at TEDD.org.

[20] The use of Instructional Activity protocols to enable teachers to teach mathematics ambitiously is based on research conducted by the Learning Teaching Practice project. See Lampert & Graziani (2009) and Lampert et al. (2010).

[21] This derives from Aristotelian ethics and psychological research on the acquisition of habits. For a contemporary perspective on this argument see Horn (2012).

REFERENCES

Abraham. A. 2003. "The Role of Visual Representations in the Learning of Mathematics." *Educational Studies in Mathematics*. Vol. 52, No. 3.

Angier, C. & Povey, H. 1999. "One Teacher and a Class of School Students: Their Perception of the Culture of their Mathematics Classroom and its Construction." *Educational Review*. Vol. 51.

Asante, K. 2012. "Secondary Students' Attitudes towards Mathematics." *IFE Psychology*. Vol. 20, No. 1.

Ball, D.L. & Cohen, D.K. 1999. "Developing Practice, Developing Practitioners: Toward a Practice-Based Theory of Professional Education." In G. Sykes & L. Darling-Hammond, ed. *Teaching as the Learning Profession: Handbook of Policy and Practice*. San Francisco, CA: Jossey Bass.

Barkatsas, A. & Malone, J. 2005. "A Typology of Mathematics Teachers' Beliefs about Teaching and Learning Mathematics and Instructional Practices." *Mathematics Education Research Journal*. Vol. 17, No. 2.

Bereiter, C. & Scardamalia, M. 1985. "Cognitive Coping and the Problem of Inert Knowledge." In S. Chipman, J. Segal, & R. Glaser, ed. *Thinking and Learning Skills: Volume 2, Research and Open Questions*. Mahwah, NJ: Erlbaum.

Blanton, M. & Kaput, J. 2005. "Teaching and Learning a New Algebra." In E. Fennema & T. Romberg, ed. *Mathematics Classrooms that Promote Understanding*. Mahwah, NJ: Erlbaum.

Boaler, J. & Greeno, J.G. 2000. "Identity, Agency, and Knowing in Mathematics Worlds." In J. Boaler, ed. *Multiple Perspectives on Mathematics Teaching and Learning*. Stamford, CT: Elsevier Science.

Bransford, J., Brown, A., & Cocking, R., eds. 2000. *How People Learn: Brain, Mind, Experience, and School*. Washington, DC: National Academy Press, National Research Council.

Bryk, A.S., Sebring, P.B., Allensworth, E., Luppescu, S., & Easton, J.Q. 2010. *Organizing Schools for Improvement: Lessons from Chicago*. Chicago, IL: University of Chicago.

Cawelti, G. 2006. "The Side Effects of NCLB." *Educational Leadership*. Vol. 64, No. 3.

Chapin, S. & O'Connor, C. 2007. "Academically Productive Talk: Supporting Students' Learning in Mathematics." In W. Gary Martin & M. Strutchens, ed. *The Learning of Mathematics, Sixty-ninth Yearbook of the National Council of Teachers of Mathematics (NCTM)*. Reston, VA: National Council of Teachers of Mathematics.

Cohen, D.K. & Spillane, J. 1992. "Policy and Practice: The Relations between Governance and Instruction." *Review of Research in Education*. Vol. 18.

Cohen, D.K. & Hill, H.C. 2001. *Learning Policy*. New Haven, CT: Yale University Press.

Cobb, P. & Jackson, K. 2010. "Towards an Empirically Grounded Theory of Action for Improving the Quality of Mathematics Teaching at Scale." *Mathematics Teacher Education and Development*. Vol. 13, No. 1.

Coburn, C.E. 2003. "Rethinking Scale: Moving beyond Numbers to Deep and Lasting Change." *Educational Researcher*. Vol. 32, No. 6.

Cohen, D.K., et al. 2013. *Improvement by Design*. Chicago, IL: University of Chicago Press.

Cohen, D.K., Raudenbusch, S., & Ball, D.L. 2003. "Resources, Instruction, and Research." *Educational Evaluation and Policy Analysis*. Vol. 25, No. 2.

Cohen, D.K. 2011. *Teaching and its Predicaments*. Cambridge, MA: Harvard University Press.

Cohen, D.K. 1988. *Teaching Practice: Plus que ca change*. East Lansing, MI: Michigan State University, National Center for Research on Teacher Education.

Colby, S. & Wicoff, K. 2006. *Aspire Public Schools: From 10 Schools to 6-Million Kids*. Boston, MA: Bridgespan Group.

Cossentino, J.M. 2005. "Ritualizing Expertise: A Non-Montessorian View of the Montessori Method." *American Journal of Education*. Vol. 111.

Cuban, L. 1993. *How Teachers Taught: Constancy and Change in American Classrooms, 1890-1990*. New York, NY: Teachers College Press.

Cusick, P. 1983. *The Egalitarian Ideal and the American High School: Studies of Three Schools*. New York, NY: Longman.

Cusick, P. 1973. *Inside High School: The Student's World*. New York, NY: Holt, Rinehart, and Winston.

Deal, T.E. & Peterson, K.D. 1999. *Shaping School Culture: The Heart of Leadership*. San Francisco, CA: Jossey-Bass.

Deal, T.E. & Peterson, K.D. 2009. *Shaping School Culture: Pitfalls, Paradoxes, & Promises*. San Francisco, CA: Jossey-Bass.

Dweck, C. 2007. *Mindset: The New Psychology of Success: How We Can Learn to Fulfill our Potential*. New York, NY: Ballantine Books.

Eckert, P. 1989. *Jocks and Burnouts: Social Categories and Identity in the High School*. New York, NY: Teachers College Press.

Eckert, P. 1990. "Adolescent Social Categories, Information and Science Learning." In M. Gardner, J. Greeno, F. Reif, & A. Schoenfeld, ed. *Toward a Scientific Practice of Science Education*. Hillsdale, NJ: Lawrence Earlbaum.

Elmore, R.F. 1996. "Getting to Scale with Good Educational Practice." *Harvard Educational Review*. Vol. 66, No. 1.

Fennema, E. & Carpenter, T.P. 1996. "A Longitudinal Study of Learning to Use Children's Thinking in Mathematics Instruction." *Journal for Research in Mathematics Education*. Vol. 27, No. 4.

Fountas, I.C. & Pinnell, G.S. 1996. *Guided Reading: Good First Teaching for All Children*. Portsmouth, NH: Heinemann.

Ghousseini, H., Beasley, H., & Lord, S. 2015. "Investigating the Potential of Guided Practice with an Enactment Tool for Supporting Adaptive Performance." *Journal of the Learning Sciences*. Vol. 24, No. 3.

Goos, M. 2004. "Learning Mathematics in a Classroom Community of Inquiry." *Journal for Research in Mathematics Education*. Vol. 35, No. 4.

Gordon, E.W. 2000. *Pedagogy and the Development of Intellective Competence*. New York, NY: College Board National Forum.

Graff, G. 2003. *Clueless in Academe: How Schooling Obscures the Life of the Mind*. New Haven, CT: Yale University Press.

Greeno, J.G. 2001. *Students with Competence, Authority and Accountability: Affording Intellective Identities in Classrooms*. New York, NY: The College Board.

Halliday, M.A.K. 1978. *Language as Social Semiotic*. London, England: Edward Arnold.

Hannula, M. 2002. "Attitude towards Mathematics: Emotions, Expectations, and Values." *Educational Studies in Mathematics*. Vol. 49, No. 1.

Hewlett Foundation. 2013. "Deeper Learning Competencies." Available at: http://www.hewlett.org/library/hewlett-foundation-publication/deeper-learning-defined

Hiebert, J. et al. 2003. *Teaching Mathematics in Seven Countries: Results from the TIMSS 1999 Video Study*. Washington, DC: National Center for Education Statistics.

Indiana University, Bloomington. n.d. "High School Survey of Student Engagement." Available at: http://ceep.indiana.edu/hssse/index.html

Holt, J. 1969. *The Underachieving School*. New York, NY: Pitman.

Horn, I. 2012. *Strength in Numbers: Collaborative Learning in Secondary Mathematics*. Reston, VA, National Council of Teacher of Mathematics.

Jackson, K., et al. 2015. "Investigating the Development of Mathematics Leaders' Capacity to Support Teachers' Learning on a Large Scale." *ZDM (International Journal of Mathematics Education)*. Vol. 47, No. 1.

Jackson, P. 1968. *Life in Classrooms*. New York, NY: Holt, Rinehart, and Winston.

Jennings, J. 2012. *Reflections on a Half-Century of School Reform: Why Have we Fallen Short and Where Do we Go from Here?* Washington, DC: Center on Education Policy.

Kazemi, E. 2008. "School Development as a Means to Improve Mathematics Teaching and Learning: Towards Multidirectional Analyses of Learning Across Contexts." In K. Krainer, & T. Wood, ed. *Participants in Mathematics Teacher Education: Individuals, Teams, Communities, and Networks*. Rotterdam, The Netherlands: Sense Publishers.

Kline, M. 1972. *Mathematical Thought from Ancient to Modern Times*. New York, NY: Oxford University Press.

Lampert, M. 2001. *Teaching Problems and the Problems of Teaching*. New Haven, CT: Yale University Press.

Lampert, M., Beasley, H., Ghousseini, H., Kazemi, E., & Franke, M. 2010. "Using Designed Instructional Activities to Enable Novices to Manage Ambitious Mathematics Teaching." In M.K. Stein & L. Kucan, ed. *Instructional Explanations in the Disciplines*. New York, NY: Springer.

Lampert, M., Boerst, T., & Graziani, F. 2011. "Organizational Assets in the Service of School-Wide Ambitious Teaching Practice." *Teachers College Record*. Vol. 113, No. 7.

Lampert, M., Franke, M., Kazemi, E., Ghousseini, H., Turrou, A.C., Beasley, H., & Crowe, K. 2013. "Keeping it Complex: Using Rehearsals to Support Novice Teacher Learning of Ambitious Teaching." *Journal of Teacher Education*. Vol. 64. No. 3.

Lampert, M. & Graziani, F. 2009. "Instructional Activities as a Tool for Teachers' and Teacher Educators' Learning." *Elementary School Journal*. Vol. 109, No. 5.

Leinhardt, G., Zaslavsky, O., & Stein, M. 1990. "Functions, Graphs, and Graphing: Tasks, Learning, and Teaching." *Review of Educational Research*. Vol. 60, No. 1.

Lipnevich, A., MacCann, C., Krumm, S., Burrus, J., & Roberts, R. 2011. "Mathematics Attitudes and Mathematics Outcomes of US and Belarusian Middle School Students." *Journal of Educational Psychology*. Vol. 103, No. 1.

Lortie, D.C. 1975. *Schoolteacher*. Chicago, IL: University of Chicago Press.

Loucks-Horsley, S. & Matsumoto, C. 1999. "Research on Professional Development for Teachers of Mathematics and Science: The State of the Scene." *School Science and Mathematics*. Vol. 99, No. 5.

McDonald, M., Kazemi, E., & Kavanagh, S. 2013. "Core Practices and Pedagogies of Teacher Education: A Call for a Common Language and Collective Activity." *Journal of Teacher Education*. Vol. 64.

Mehta, J. & Fine, S. 2015. *The Why, What, Where, and How of Deeper Learning in American Secondary Schools*. Students at the Center: Deeper Learning Research Series. Boston, MA: Jobs for the Future.

Moses, R. & Cobb, C. 2001. *Radical Equations: Civil Rights from Mississippi to the Algebra Project*. Boston, MA: Beacon Press.

National Council of Teachers of Mathematics. 2014. *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Washington, DC: Author

National Mathematics Advisory Panel. 2008. *Foundations for Success: The Final Report of the National Mathematics Advisory Panel.* Washington, DC: U.S. Department of Education.

National Research Council. 2005. *How Students Learn: History, Mathematics, and Science in the Classroom*. Committee on *How People Learn*, A Targeted Report for Teachers. M.S. Donovan & J.D. Bransford, eds. Washington, DC: The National Academies Press, Division of Behavioral and Social Sciences and Education.

Newmann, F., & Associates. 1996. *Authentic Achievement: Restructuring Schools for Intellectual Quality*. San Francisco, CA: Jossey-Bass.

Polya, G. 1954. *Mathematics and Plausible Reasoning*. Princeton, NJ: Princeton University Press.

Powell, A., Farrar, E., & Cohen, D. 1985. *The Shopping Mall High School*. Boston, MA: Houghton Mifflin.

Ritchhardt, R. 1997. "Seeing the World through Mathematical Eyes." In R. Richhart, ed. Through *Mathematical Eyes: Exploring Functional Relationships in Math and Science*. Portsmouth, NH: Heinemann.

Roeser, R., Eccles, J., & Sameroff, A. 2000. "School as a Context of Early Adolescents' Academic and Social-Emotional Development: A Summary of Research Findings." *Elementary School Journal*. Vol. 100, No. 5.

Rogoff, B. 1990. *Apprenticeship in Thinking: Cognitive Development in Social Context*. New York, NY: Oxford University Press.

Rogoff, B., Baker-Sennet, J., Lacasa, P., & Goldsmith, D. 1995. "Development through Participation in Sociocultural Activity." *New Directions for Child Development*. Vol. 67.

Rosenberg, S. 2012. *Organizing for Quality in Education: Individualistic and Systemic Approaches to Teacher Quality*. Ann Arbor, MI: University of Michigan.

Sam, L.C. & Ernest, P. 2000. "A Survey of Public Images of Mathematics." *Research in Mathematics Education*. Vol. 2, No. 1.

Sanchez, K., Zimmerman, B., & Ye, R. 2004. "Secondary Students' Attitudes toward Mathematics." *Academic Exchange Quarterly*. Vol. 8, No. 2.

Sfard, A. 2008. *Thinking as Communicating: Human Development, the Growth of Discourses, and Mathematizing*. Cambridge, MA: Cambridge University Press.

Sims, L. 2008. "Look who's Talking: Differences in Math Talk in U.S. and Chinese Classrooms." *Teaching Children Mathematics*. Vol. 15, No. 2.

Smith, M. & Stein, M.K. 2011. *Five Practices for Orchestrating Productive Mathematics Discussions*. Reston, VA: National Council of Teachers of Mathematics.

Staples, M., Bartlo, J., & Thanheiser, E. 2012. "Justification as a Teaching and Learning Practice: Its (Potential) Multifaceted Role in Middle Grades Mathematics Classrooms." *Journal of Mathematical Behavior*. Vol. 31.

Staples, M. & Truxaw, M. 2010. "The Mathematics Learning Discourse Project: Fostering Higher Order Thinking and Academic Language in Urban Mathematics Classrooms." *Journal of Urban Mathematics Education*. Vol. 3, No. 1.

Stein, M.K., Russell, J., & Smith, M. 2011. "The Role of Tools in Bridging Research and Practice in an Instructional Improvement Effort." In W. Tate, K.D. King, & C. Anderson, ed. *Disrupting Tradition: Research and Practice Pathways in Mathematics Education*. Reston, VA: National Council of Teachers of Mathematics.

Stevens, R. & Hall, R. 1998. "Disciplined Perception: Learning to See in Technoscience." In M. Lampert & M. Blunk, eds. *Talking Mathematics in School*. Cambridge, MA: Cambridge University Press.

Stigler, J.W. & Thompson, B. 2009. "Thoughts on Creating, Accumulating, and Utilizing Shareable Knowledge to Improve Teaching." *Elementary School Journal*. Vol. 109, No. 5.

Swan, M. 2005. *Improving Learning in Mathematics: Challenges and Strategies*. Sheffield, UK: Teaching and Learning Division, Department for Education and Skills Standards.

U.S. Department of Education, National Center for Education Statistics. 2012. *The Condition of Education 2012*. Washington, DC: National Center for Education Statistics.

U.S. Department of Education, National Center for Education Statistics. 2015. "Integrated Postsecondary Education Data System. Fall 2009 through Fall 2013, Completions component." Available at: https://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2014067

U.S. Department of Education, National Center for Education Statistics. 2012. "Common Core of Data (CCD), NCES Common Core of Data State Dropout and Graduation Rate Data file, School Year 2011-12, Preliminary Version."

Waller, W. 1932. *The Sociology of Teaching*. New York, NY: John Wiley & Sons.

Weick, K. & McDaniel, R. 1989. "How Professional Organizations Work: Implications for School Organization and Management." In T.J. Sergiovanni & J.H. Moore, ed. *Schooling for Tomorrow: Directing Reforms to Issues that Count*. Rockleigh, NJ: Allyn & Bacon.

Wenger, E. 1998. *Communities of Practice: Learning, Meaning, and Identity*. Cambridge, MA: Cambridge University Press.

Wertsch, J. 1988. *Vygotsky and the Social Formation of Mind*. Cambridge, MA: Harvard University Press.

Wertsch, J. 1985. *Culture, Communication, and Cognition: Vygotskian Perspectives*. Cambridge, MA: Cambridge University Press.

Whitehead, A.N. 1929. *The Aims of Education and Other Essays*. New York, NY: Macmillan.

Wiliam, D. 2007. "Content then Process: Teacher Learning Communities in the Service of Formative Assessment." In D. Reeves, ed. *Ahead of the Curve: The Power of Assessment to Transform Teaching and Learning*. Bloomington, IN: Solution Tree Press.

Wortham, S.E. 2004. "The Interdependence of Social Identification and Learning." *American Educational Research Journal*. Vol. 41, No. 3.

Zimmerman, B. 2001. "Theories of Self-Regulated Learning and Academic Achievement: An Overview and Analysis." In B. Zimmerman & D. Schunk, ed. *Self-Regulated Learning and Academic Achievement: Theoretical Perspectives*. Mahwah, NJ: Erlbaum.

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